Chapter 03
Number Systems

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In this chapter you will learn about:

- Non-positional number system
- Positional number system
- Decimal number system
- Binary number system
- Octal number system
- Hexadecimal number system

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Learning Objectives

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- Convert a number’s base
  - Another base to decimal base
  - Decimal base to another base
  - Some base to another base
- Shortcut methods for converting
  - Binary to octal number
  - Octal to binary number
  - Binary to hexadecimal number
  - Hexadecimal to binary number
- Fractional numbers in binary number system
Two types of number systems are:

- Non-positional number systems
- Positional number systems
Non-positional Number Systems

**Characteristics**

- Use symbols such as I for 1, II for 2, III for 3, IIII for 4, IV for 5, etc
- Each symbol represents the same value regardless of its position in the number
- The symbols are simply added to find out the value of a particular number

**Difficulty**

- It is difficult to perform arithmetic with such a number system
Positional Number Systems

- **Characteristics**
  - Use only a few symbols called digits
  - These symbols represent different values depending on the position they occupy in the number

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Positional Number Systems

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- The value of each digit is determined by:
  1. The digit itself
  2. The position of the digit in the number
  3. The base of the number system

(base = total number of digits in the number system)

- The maximum value of a single digit is always equal to one less than the value of the base
### Characteristics

- A positional number system
- Has 10 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). Hence, its base = 10
- The maximum value of a single digit is 9 (one less than the value of the base)
- Each position of a digit represents a specific power of the base (10)
- We use this number system in our day-to-day life

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Decimal Number System

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Example

\[ 2586_{10} = (2 \times 10^3) + (5 \times 10^2) + (8 \times 10^1) + (6 \times 10^0) \]

\[ = 2000 + 500 + 80 + 6 \]
Characteristics

- A positional number system
- Has only 2 symbols or digits (0 and 1). Hence its base = 2
- The maximum value of a single digit is 1 (one less than the value of the base)
- Each position of a digit represents a specific power of the base (2)
- This number system is used in computers

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Binary Number System

(Continued from previous slide.)

Example

\[ 10101_2 = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) \times (1 \times 2^0) \]

\[ = 16 + 0 + 4 + 0 + 1 \]

\[ = 21_{10} \]
In order to be specific about which number system we are referring to, it is a common practice to indicate the base as a subscript. Thus, we write:

\[ 10101_2 = 21_{10} \]
Bit stands for binary digit

A bit in computer terminology means either a 0 or a 1

A binary number consisting of \( n \) bits is called an \( n \)-bit number
Characteristics

- A positional number system
- Has total 8 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7). Hence, its base = 8
- The maximum value of a single digit is 7 (one less than the value of the base)
- Each position of a digit represents a specific power of the base (8)

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Octal Number System

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Since there are only 8 digits, 3 bits \(2^3 = 8\) are sufficient to represent any octal number in binary.

**Example**

\[
2057_8 = (2 \times 8^3) + (0 \times 8^2) + (5 \times 8^1) + (7 \times 8^0)
\]

\[
= 1024 + 0 + 40 + 7
\]

\[
= 1071_{10}
\]
Hexadecimal Number System

Characteristics

- A positional number system
- Has total 16 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F). Hence its base = 16
- The symbols A, B, C, D, E and F represent the decimal values 10, 11, 12, 13, 14 and 15 respectively
- The maximum value of a single digit is 15 (one less than the value of the base)

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Each position of a digit represents a specific power of the base (16)

Since there are only 16 digits, 4 bits \(2^4 = 16\) are sufficient to represent any hexadecimal number in binary

**Example**

\[1AF_{16} = (1 \times 16^2) + (A \times 16^1) + (F \times 16^0)\]

\[= 1 \times 256 + 10 \times 16 + 15 \times 1\]

\[= 256 + 160 + 15\]

\[= 431_{10}\]
Converting a Number of Another Base to a Decimal Number

Method

Step 1: Determine the column (positional) value of each digit

Step 2: Multiply the obtained column values by the digits in the corresponding columns

Step 3: Calculate the sum of these products
Converting a Number of Another Base to a Decimal Number

(Continued from previous slide..)

**Example**

\[ 4706_8 = ?_{10} \]

\[ 4706_8 = 4 \times 8^3 + 7 \times 8^2 + 0 \times 8^1 + 6 \times 8^0 \]

\[ = 4 \times 512 + 7 \times 64 + 0 + 6 \times 1 \]

\[ = 2048 + 448 + 0 + 6 \]

\[ = 2502_{10} \]

Common values multiplied by the corresponding digits

Sum of these products
Division-Remainder Method

Step 1: Divide the decimal number to be converted by the value of the new base

Step 2: Record the remainder from Step 1 as the rightmost digit (least significant digit) of the new base number

Step 3: Divide the quotient of the previous divide by the new base
Converting a Decimal Number to a Number of Another Base

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Step 4: Record the remainder from Step 3 as the next digit (to the left) of the new base number

Repeat Steps 3 and 4, recording remainders from right to left, until the quotient becomes zero in Step 3

Note that the last remainder thus obtained will be the most significant digit (MSD) of the new base number

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Converting a Decimal Number to a Number of Another Base

(Continued from previous slide..)

Example

\[ 952_{10} = ?_8 \]

Solution:

\[
\begin{array}{c|c|c}
8 & 952 & \text{Remainder} \\
8 & 119 & \text{S} \\
8 & 14 & 7 \\
8 & 1 & 6 \\
8 & 0 & 1 \\
\end{array}
\]

Hence, \[ 952_{10} = 1670_8 \]
Converting a Number of Some Base to a Number of Another Base

**Method**

1. Convert the original number to a decimal number (base 10)
2. Convert the decimal number so obtained to the new base number

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Example

\[ 545_6 = ?_4 \]

Solution:

Step 1: Convert from base 6 to base 10

\[
 545_6 = 5 \times 6^2 + 4 \times 6^1 + 5 \times 6^0 \\
= 5 \times 36 + 4 \times 6 + 5 \times 1 \\
= 180 + 24 + 5 \\
= 209_{10}
\]
Step 2: Convert $209_{10}$ to base 4

<table>
<thead>
<tr>
<th>4</th>
<th>209</th>
<th>Remainders</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>52</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

Hence, $209_{10} = 3101_4$

So, $545_6 = 209_{10} = 3101_4$

Thus, $545_6 = 3101_4$
Shortcut Method for Converting a Binary Number to its Equivalent Octal Number

Method

Step 1: Divide the digits into groups of three starting from the right

Step 2: Convert each group of three binary digits to one octal digit using the method of binary to decimal conversion

(Continued on next slide)
Example

1101010_2 = ?_8

Step 1: Divide the binary digits into groups of 3 starting from right

001 101 010

Step 2: Convert each group into one octal digit

001_2 = 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1
101_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 5
010_2 = 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 2

Hence, 1101010_2 = 152_8
Method

Step 1: Convert each octal digit to a 3 digit binary number (the octal digits may be treated as decimal for this conversion)

Step 2: Combine all the resulting binary groups (of 3 digits each) into a single binary number

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Shortcut Method for Converting an Octal Number to Its Equivalent Binary Number

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Example

$562_8 = ?_2$

Step 1: Convert each octal digit to 3 binary digits

$5_8 = 101_2, \quad 6_8 = 110_2, \quad 2_8 = 010_2$

Step 2: Combine the binary groups

$562_8 = \overline{101} \quad 110 \quad 010$

Hence, $562_8 = 101110010_2$
Shortcut Method for Converting a Binary Number to its Equivalent Hexadecimal Number

**Method**

**Step 1:** Divide the binary digits into groups of four starting from the right

**Step 2:** Combine each group of four binary digits to one hexadecimal digit

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Shortcut Method for Converting a Binary Number to its Equivalent Hexadecimal Number

(Continued from previous slide..)

Example

\[ 111101_2 = ?_{16} \]

Step 1: Divide the binary digits into groups of four starting from the right

\[
\begin{align*}
0011 & \quad 1101 \\
\end{align*}
\]

Step 2: Convert each group into a hexadecimal digit

\[
\begin{align*}
0011_2 & = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3_{10} = 3_{16} \\
1101_2 & = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 3_{10} = D_{16} \\
\end{align*}
\]

Hence, \[ 111101_2 = 3D_{16} \]
Shortcut Method for Converting a Hexadecimal Number to its Equivalent Binary Number

Method

Step 1: Convert the decimal equivalent of each hexadecimal digit to a 4 digit binary number

Step 2: Combine all the resulting binary groups (of 4 digits each) in a single binary number

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Shortcut Method for Converting a Hexadecimal Number to its Equivalent Binary Number

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Example

$2\text{AB}_{16} = ?_2$

Step 1: Convert each hexadecimal digit to a 4 digit binary number

$2_{16} = 2_{10} = 0010_2$
$A_{16} = 10_{10} = 1010_2$
$B_{16} = 11_{10} = 1011_2$
Shortcut Method for Converting a Hexadecimal Number to its Equivalent Binary Number

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Step 2: Combine the binary groups

\[
2\text{AB}_{16} = \begin{array}{c}
0010 \\
1010 \\
1011 \\
\end{array}
\]

\[
2 \quad \text{A} \quad \text{B}
\]

Hence, \(2\text{AB}_{16} = 001010101011_2\)
Fractional numbers are formed same way as decimal number system

In general, a number in a number system with base $b$ would be written as:
$$a_n a_{n-1} \ldots a_0 \cdot a_{-1} a_{-2} \ldots a_{-m}$$

And would be interpreted to mean:
$$a_n \times b^n + a_{n-1} \times b^{n-1} + \ldots + a_0 \times b^0 + a_{-1} \times b^{-1} + a_{-2} \times b^{-2} + \ldots + a_{-m} \times b^{-m}$$

The symbols $a_n, a_{n-1}, \ldots, a_{-m}$ in above representation should be one of the $b$ symbols allowed in the number system.
### Formation of Fractional Numbers in Binary Number System (Example)

<table>
<thead>
<tr>
<th>Position</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>-1</th>
<th>-2</th>
<th>-3</th>
<th>-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position Value</td>
<td>(2^4)</td>
<td>(2^3)</td>
<td>(2^2)</td>
<td>(2^1)</td>
<td>(2^0)</td>
<td>(2^{-1})</td>
<td>(2^{-2})</td>
<td>(2^{-3})</td>
<td>(2^{-4})</td>
</tr>
<tr>
<td>Quantity Represented</td>
<td>16</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{4})</td>
<td>(\frac{1}{8})</td>
<td>(\frac{1}{16})</td>
</tr>
</tbody>
</table>

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Example

\[110.101_2 = 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}\]
\[= 4 + 2 + 0 + 0.5 + 0 + 0.125\]
\[= 6.625_{10}\]
## Formation of Fractional Numbers in Octal Number System (Example)

<table>
<thead>
<tr>
<th>Position</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>-1</th>
<th>-2</th>
<th>-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position Value</td>
<td>$8^3$</td>
<td>$8^2$</td>
<td>$8^1$</td>
<td>$8^0$</td>
<td>$8^{-1}$</td>
<td>$8^{-2}$</td>
<td>$8^{-3}$</td>
</tr>
<tr>
<td>Quantity Represented</td>
<td>512</td>
<td>64</td>
<td>8</td>
<td>1</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{64}$</td>
<td>$\frac{1}{512}$</td>
</tr>
</tbody>
</table>

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Formation of Fractional Numbers in Octal Number System (Example)

(Continued from previous slide..)

Example

\[
127.54_8 = 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 5 \times 8^{-1} + 4 \times 8^{-2}
= 64 + 16 + 7 + \frac{5}{8} + \frac{4}{64}
= 87 + 0.625 + 0.0625
= 87.6875_{10}
\]
Key Words/Phrases

- Base
- Binary number system
- Binary point
- Bit
- Decimal number system
- Division-Remainder technique
- Fractional numbers
- Hexadecimal number system
- Least Significant Digit (LSD)
- Memory dump
- Most Significant Digit (MSD)
- Non-positional number system
- Number system
- Octal number system
- Positional number system